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Diameter Structure Analysis of Forest Stand and Selection of Suitable Model

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Abstract

Ecologically and economically it is important to understand how many tree stems are in each diameter class. The purpose of this study was to find larch forest (*Larix sibirica*) diameter distribution model among Weibull, Burr and Johnson SB distributions. Inventory was conducted near Gachuurt village, Ulaanbaatar, Mongolia. The goodness of fit test were accompanied with Kolmogorov-Smirnov, Anderson-Darling and Chi-Squared tests for distribution models. Study result shows Johnson SB distribution gave the best performance in terms of quality of fit to the diameter distribution of larch forest.

Key words: diameter distribution model, Weibull, Burr, Johnson SB, larch

Introduction

Detailed information of forest stand is crucial for forest research and planning. This information used for input of ecosystem modeling and/or forest growth and yield models. In the analysis of stand dynamics, detailed data for all trees on a plot is often lacking. In such case, we may generate missing data using various theoretical diameter (D) distributions. For many years there were various activity and interest in describing the frequency distribution of D measurements in forest stands using probability density functions. First study of D distribution mathematical description was negative exponential (DeLiocourt 1898), and since then, researchers used various distributions.

All distribution models have their advantage and sensitive in specific shape. Weibull distribution able to describe Exponential, Normal and Lognormal distribution shapes (Bailey & Dell, 1973; Lin *et al.,* 2007), while Burr distribution cover much larger area of skewness and kurtosis plane than the Weibull distribution (Lindsay *et al.,* 1996). Moreover, it is closely approximate with above mentioned distributions plus Gamma, Logistic and several Pearson type distributions. Johnson SB distribution cover different region of skewness and kurtosis plane than the Burr (Johnson, 1949; Hafley & Schreuder, 1977), and it is closely approximate Beta and generalized Weibull distributions.

In case of Mongolian forests, Khongor *et al.* (2011a) published the birch forest D study using Weibull and Lognormal distributions and compared the accurateness of these models. For larch forest D distribution, Khongor *et al.,* (2011b) used Exponential, Lognormal and Gaussian (or Normal) distributions, but they did not used Weibull, Burr and Johnson SB before.

The purpose of this study is to investigate the suitability of the Weibull, Burr and Johnson SB distributions for modeling D distribution of larch forest (*Larix sibirica*).

Weibull distribution

One of the most popular models is the Weibull distribution, first introduced to the forestry research field by Bailey and Dell (1973). The popularity of the Weibull distribution depends largely on its simplicity and yet relatively good flexibility. It describes the inverse J shape for $\alpha<1$ and the exponential distribution for $\alpha=1$. For $1<\alpha<3.6$ the density function is mound shaped and positively skewed and for $\alpha=3.6$ the density function becomes approximately normal. If $\alpha>3.6$ the density function becomes increasingly negatively skewed. With the support random variable x: $\gamma \le x < +\infty$ the Probability Density Function (pdf) of Weibull 3 parameter distribution is given as:

$$f(x) = \frac{\alpha}{\beta}\left(\frac{x-\gamma}{\beta}\right)^{\alpha-1} \exp\left(\left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right) \quad (1)$$

where, α > 0 - shape parameter, β > 0 - scale parameter, γ - location parameter if γ=0, then the distribution is 2 parameter.

Burr distribution

The Burr distribution was introduced to the forestry research by Lindsay *et al*. (1996). This distribution is inherently more flexible, because it covers a much larger area of the skewness-kurtosis plane than the Weibull distribution (Lindsay *et al*., 1996; Rodriguez, 1977; Tadikamalla, 1980).

The Burr (Zimmer & Burr, 1963) distribution has a flexible shape, controllable scale and location, which makes it appealing to fit to data. It is sometimes considered as an alternative to a Normal distribution when data show slight positive skewness. With the support random variable x: $\gamma \le x < +\infty$, the pdf of Burr 4 parameter distribution is given as:

$$f(x) = \frac{\alpha k\left(\frac{x-\gamma}{\beta}\right)^{\alpha-1}}{\beta\left(1+\left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right)^{k+1}} \quad (2)$$

where, k, α > 0 - two shape parameters, β > 0 - scale parameter, γ - location parameter if γ=0, then the distribution is Burr 3 parameter.

Johnson SB distribution

The Johnson SB (1949) have been much commonly used in forest distributional studies (Hafley & Schreuder, 1977), because of its flexibility of distributional form and its ability to represent equally well positive and negative skewed distributions. The pdf of SB distribution transforms a bounded random variable by subtracting the minimum and dividing by the range. The logit of this transformation is then distributed as a standard normal variable. Following Johnson, consider this transform z on the random variable x:

$$z = \frac{x-\xi}{\lambda} \quad (3)$$

where, ξ - minimum value of x, $\xi + \lambda$ - maximum value of x.

$$\xi \le x \le \xi + \lambda \quad (4)$$

Within our context x is a D measurement. Then the pdf of D is defined as

$$f(x) = \frac{\delta}{\lambda\sqrt{2\pi}z(1-z)} \exp\left(-\frac{1}{2}\left(\gamma + \delta \ln\left(\frac{z}{1-z}\right)\right)^2\right) \quad (5)$$

where, γ and δ shape (δ > 0), λ scale (λ > 0) and ξ location parameter.

Materials and Methods

Field measurement. Study plot was selected near the Gachuurt village in the vicinity of Ulaanbaatar city, Mongolia, located at 48°00'18.9''N and 107°13'23.1' E with altitudinal elevation 1607-1627 m above sea level. The forest consisted of natural stands and any management activity had taken previously. Inventory was conducted in summer of 2009. Composition of the stands is pure larch. Plot size was 0.2 ha, i.e. 40 x 50 m in area. D were measured for all trees >1.3 m, and totally 275 stems were counted. Average D of tree stands in plot was 14.6 cm with standard error mean 0.497 cm. Diameter of tree stems ranges from 2 to 32 cm. D distribution skewness value was 0.38 indicating that the tail on the right side of the pdf is longer than the left side and kurtosis value -0.97 indicating statistically flattered peak.

Data analysis. The goodness of fit of empirical D distribution was tested using three theoretical distributions: Johnson SB, Weibull and Burr. The distribution parameters were estimated using the EasyFit 5.5 Professional distribution fitting software (Table 2). To calculate goodness-of-fit of the actual D and height distributions with theoretical distributions, the KS (Kolmogorov-Smirnov) test, χ^2 test and AD (Anderson-Darling) test (Anderson & Darling, 1952) were used. KS test is distribution free and based on empirical distribution. It is used for continuous distributions and compares curves maximum distance. It is more sensitive near the center of distribution than at the tails. The χ^2 test divides the range of the data into a set of equiprobable classes. AD test is a statistical test of whether there is evidence that a given sample of data did not arise from a given probability distribution. In its basic form, the test assumes that there are no parameters to

Table 1. Parameter estimates of the three distribution models for the study plot

Johnson SB				Weibull		Burr		
γ	δ	λ	ξ	α	β	k	α	β
0.38167	0.68757	31.863	1.81	1.8296	16.335	1474.1	1.8545	841.49

Table 2. Summary of empirical diameter distribution for larch forest (α=0.05)

Distribution	Kolmogorov Smirnov (critical value 0.08189)		Anderson Darling (critical value 2.5018)	Chi-Squared (critical value 15.507)	
	statistic	*P*-value	statistic	statistic	*P*-value
Johnson SB	0.03106	0.94589	0.18795	7.6044	0.47303
Weibull	0.06891	0.14004	1.8136	10.535	0.22946
Burr	0.07506	0.08567	1.909	13.562	0.09392

be estimated in the distribution being tested, in which case the test and its set of critical values is distribution-free.

Results

The parameter estimates of the three models are given in Table 1. The predictions from each model were compared with observed frequencies. The KS, AD and χ^2 tests and P value for KS and χ^2 tests were computed for each model (Table 2). All tested distribution models were statistically fitted with observed diameter distribution and among them Johnson SB distribution was more flexible than Weibull and Burr distributions.

By the definition, the area under the pdf graph must equal 1, so the theoretical pdf values have to be multiplied by the total number of stems to match the histogram and the D coverage of bin width to calculate the number of stems in each D class.

Tree stems are smoothly distributed in diameter classes and it is statistically unimodal.

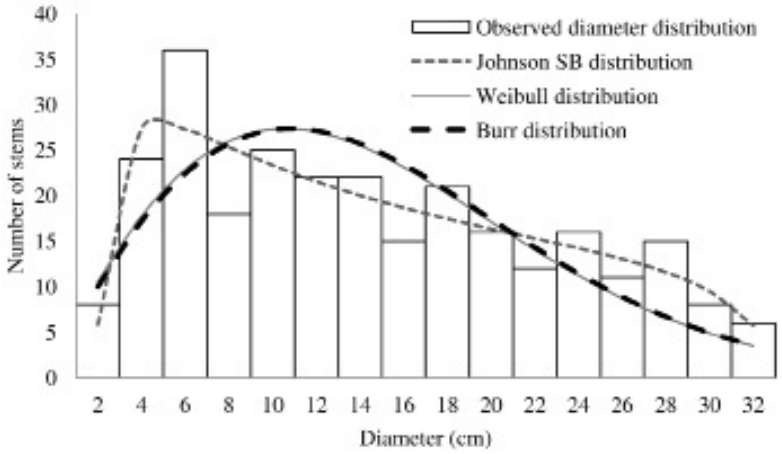

Figure 1. Model comparison for the study plot.

It is easy to fit such distribution, but here flattered peak is problem that causes under/ over prediction. Though all models passed on goodness of fit test, Weibull and Burr models over-predict D classes around 10-16 cm and lower-predict 4-6 cm and 24-30 cm classes. It is evident that the Johnson SB model was more flexible in fitting flattered D distribution of larch forest stand (Fig. 1).

Discussion

Weibull and Burr theoretical distributions fit the best for right tailed D distributions whilst Johnson SB distribution has ability to represent equally well right and left tailed distributions. With this reason we have chosen the Weibull, Burr and Johnson SB distributions to test their suitability and flexibility for larch forest D

distribution. Then, our study result suggest Johnson SB distribution for larch forest and that is would not be necessary to believe about the Johnson SB distribution is the best for all over larch forest in Mongolia.

Every forest stand D distributions are different depending on the site quality, climatic condition and history of natural or human disturbances. Supposedly, empirical distribution models would accurately work in big scale if the geographic and climate conditions are same. But, random disturbances, such as forest fire, insect invasion or selective logging are change the forest structure and shape in different forms. Specially, every forest ever influenced with forest fire in Mongolia and near urbanized areas all forests under danger of illegal timber logging.

However, it is still important that stand specific forest structure information for model development and research or management planning in small scale forest area. If we needed bigger scale as regional forest D structure, we have to collect more stand D data to fit general D distribution. The required amount of stem numbers or sample plots for regional D distribution study would be defined by stability of a chosen model. If the one fails we need to collect more stand samples and do it again until it become statistically stable. Westphal (2006) suggested that for the regional scale diameter distribution is reverse J shaped because of many small stems and relatively fewer big stems.

Strong intensity disturbances or high intensity regeneration may change D structure as bimodal. If disturbance happened repeatedly in same forest, then the D distribution would forms multimodal shape. In this study, we used unimodal D distribution. However, it may not be sufficient when a frequency distribution is reverse J with hump, bimodal or multimodal, and therefore, irregular shaped D distributions should have tested by mixture distribution (Zhang & Liu, 2006).

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